Reply to "Comment on 'Earthquake cycles and physical modeling of the process leading up to a large earthquake'"

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In response to Cocco and Bizzarri's (2002) argument that "*there is no need to assume that friction must become independent of slip rate at high speeds to resemble slip weakening*", I remarked in my previous paper (Ohnaka, 2004) that "*this argument seems logically inconsistent, because the effect of high velocity cutoff has been incorporated into the equation used in their simulation.*" The most recent comment by Bizzarri and Cocco (2006) represents their counterargument to this remark. The basis of their reasoning is that even if the +1 term was included in the argument of the logarithm in equations (1c) and (1d) described in the paper (Ohnaka, 2004), the traction dependence on slip rate would remain in the governing equation, because the state variable depends on the fault slip velocity (Bizzarri and Cocco, 2006).

I feel the need to point out, however, that their counterargument still appears to be logically inconsistent. In their simulations of the dynamic rupture regime within the framework of the rate- and state-dependent formulation, they used the equation from which the direct effect of high slip rates on the shear traction had been cut off, which they admit in their comment (Bizzarri and Cocco, 2006). In addition, they also used the equation from which the effect of slip rate on state evolution had been removed; it would seem they are unaware of this fact.

The state variable Θ has the dimensions of time and is interpreted to be the age of the load supporting contacts across the fault surface (see Dieterich and Kilgore, 1996). According to Dieterich and Kilgore (1996), the evolution of state at constant slip speed under constant effective normal stress is given by

$$\Theta = \frac{d_c}{\dot{D}} - \left[\frac{d_c}{\dot{D}} - \Theta_0\right]\exp\left(-\frac{D}{d_c}\right) = \frac{d_c}{\dot{D}}\left[1 - \exp\left(-\frac{D}{d_c}\right)\right] + \Theta_0 \exp\left(-\frac{D}{d_c}\right) \qquad (1)$$

where Θ_0 is state at $\dot{D} = 0$; for the notation of the other parameters, refer to Ohnaka (2004). In their calculation of dynamic rupture regime under the rate- and state-dependent formulation, Cocco and Bizzarri (2002) assumed that $\Theta \gg d_c/\dot{D}$ without paying attention to what the state variable physically means; they then used the equation of $\Theta = \Theta_0 \exp(-D/d_c)$ in place of Eq. (1). Note, however, that $d_c/\dot{D}$ is the only mathematical expression of the effect of slip rate on state evolution in Eq. (1). Therefore, the assumption of $d_c/\dot{D} = 0$ physically means that no state evolution effect of slip rate is incorporated into the equation $\Theta = \Theta_0 \exp(-D/d_c)$ used in their simulations. In other words, the state variable Θ no longer depends on the fault slip velocity because the effect of slip rate on state evolution has completely been removed from the equation used in their simulations. For this reason, I believe no one can accept their counterargument. Bizzarri and Cocco (2006) argue that "*the effect of the +1 terms does not affect the traction evolution when* v_* *is greater than 0.1 m/s*". Even so, they have to use Eq. (1) without assuming that $d_c/\dot{D} = 0$ if they claim that "*the traction dependence on slip rate would remain in the governing equation because the state variable depends on the fault slip velocity*".

The rate- and state-dependent constitutive law was derived by Dieterich and Ruina (for references, see Ohnaka, 2004) based on laboratory experiments on extremely low slip rates (of the order of 1 mm/year up to 1 mm/s; for the references, see Dieterich and Kilgore, 1996), from nearly a steady state value to another steady state value; as such, the law does not warrant its extension to high slip velocities of the order of 1 m/s or higher experienced during actual dynamic rupture processes. However, Cocco and Bizzarri (2002) and Bizzarri *et al.* (2001) have arbitrarily extended mathematically this law to include such high slip velocities in dynamic rupture regimes—without any consideration of the underlying physics. Such steps must be avoided in order for the physics of earthquakes to be a quantitative science.

Bizzarri and Cocco (2006) emphasize the validity of their simulations of a dynamic rupture regime under the rate- and state-dependent constitutive formulation on the basis of the similarity between the traction evolution, with slip inferred in their numerical simulations, and the experimental results displayed by Ohnaka and Yamashita (1989). However, both the effect of slip rate on shear traction and the effect of slip rate on state evolution at high slip velocities have been removed from what they call "*the rate- and state-dependent law*" used in their simulations. Note therefore that such a

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law is no longer "*rate- and state-dependent*" at high slip velocities.

As discussed in earlier papers (Ohnaka, 2003, 2004; Ohnaka and Kato, 2006), real faults embedded in the seismogenic crust are inherently inhomogeneous, and fault inhomogeneity has profound implications for the rational constitutive formulation for earthquake ruptures. The earthquake rupture process at shallow crustal depths is not a simple one of frictional slip failure on a precut weak fault but, rather, a more complex process, including the fracture of initially intact rock at local strong areas (called "asperities") on an inhomogeneous fault (see Ohnaka and Kato, 2006). The constitutive law for real earthquake ruptures must, therefore, be formulated as a unifying law that governs not only frictional slip failure at precut-interface (or frictional contact) areas on faults but also shear fracture of intact rock at local, strong areas on the faults. This is a very important requirement to be met when we rationally formulate the constitutive law for real earthquake ruptures (see Ohnaka, 2004; Ohnaka and Kato, 2006).

However, the rate- and state-dependent formulation does not lead to such a unifying law as stated above because the rate- and state-dependent law is not applicable to the instability or stability of the shear fracture process of intact rock. I therefore stated in my previous paper (Ohnaka, 2004) that "*the rate- and state-dependent formulation does not lead to a unifying constitutive law that governs both frictional slip failure and the shear fracture of intact rock*". This statement, however, is misunderstood in the comment by Bizzarri and Cocco (2006).

Laboratory experiments (Ohnaka, 2003) have demonstrated that the breakdown process of the shear fracture of intact rock can be completely unified with that of frictional slip failure on precut faults within the framework of slip-dependent constitutive formulation, and this is corroborated by the theory of micro-contact physics of a sliding interface in intimate contact (see Ohnaka, 1996). It is therefore obvious from physical viewpoints how the constitutive law for real earthquake ruptures should be formulated, and what it ought to be (see Ohnaka, 2004; Ohnaka and Kato, 2006).

In regard to quantitative scaling of the scale-dependent physical quantities inherent in the rupture, one has to recognize that the rate- and state-dependent constitutive formulation poses a serious problem. The only possible scaling parameter incorporated into the rate- and state-dependent law is the critical slip distance d_c, which is defined as the slip distance required for friction to change from a steady-state value at a given specific slip rate $\dot{D}_1$ (for example, 10^{-4} mm/s) to a different steady-state value at another given specific slip rate $\dot{D}_2$ (for example, 10^{-3} mm/s). Note therefore that the parameter d_c is an artifact definable only in the laboratory and that the amount of d_c defined as such is not related to the geometric length of the coherent zone of rupture breakdown. For this reason it is obvious that d_c does not represent any physically meaningful scaling property inherent in the rupture breakdown.

In contrast, the breakdown displacement D_c defined within the framework of slip-dependent constitutive formulation is the slip displacement at the end of the breakdown process, and D_c is directly related to the geometric length X_c of the coherent zone of rupture breakdown (see Ohnaka and Yamashita, 1989; Ohnaka, 2000). It has been shown theoretically and experimentally (Ohnaka and Shen, 1999; Ohnaka, 2000, 2003, 2004) that scale-dependent physical quantities inherent in the rupture are scaled by D_c and that scale-dependence of the scale-dependent physical quantities is completely ascribed to the scale-dependence of D_c. Therefore, D_c defined in the framework of the slip-dependent formulation is the most appropriate scaling parameter for physical scaling of scale-dependent quantities inherent in the rupture.

I have argued above that d_c does not represent any physically meaningful scaling property inherent in the rupture breakdown. One might make a counterargument in this respect, because d_c can be related to D_c under certain assumptions. It is true that d_c is a fraction of D_c. For instance, Cocco and Bizzarri (2002) report that $D_c/d_c = 15$ on the basis of their own numerical simulation. One must recognize, however, that there is no warrant that D_c/d_c has a universal constant value of 15; this is obvious from the definition of d_c. Thus, d_c is not an appropriate parameter for quantitative scaling of the scale-dependent physical quantities inherent in the rupture. In addition, it is difficult (or impossible) to estimate d_c directly from seismological data, given the fact that d_c is an artifact definable only in the laboratory. Indeed, Guatteri *et al.* (2001) had no choice but to estimate d_c for the 1995 Kobe earthquake under certain assumptions, not straightforwardly but indirectly from the D_c estimated by Ide and Takeo (1997). This is also in favor of the slip-dependent constitutive formulation rather than the rate- and state-dependent formulation.

Bizzarri and Cocco (2006) point out that different physical mechanisms may occur in actual fault zones; as examples of such mechanisms they suggest: intact rock fracturing, comminution and pulverization, gouge creation and fragmentation, frictional heating and melting, and mechanical and chemical effects of interstitial pore fluid, including hydrodynamic lubrication and thermal pressurization, inelastic deformation of the damage zone and permeability and porosity evolution (for references, see Bizzarri and Cocco, 2006). They argue that "*all of these should be considered in the formulation of a unifying constitutive model that describes the evolution of the slipping surface*" (Bizzarri and Cocco, 2006). I do not object to this argument, although we should distinguish the most fundamental and primary property from the properties of secondary (or minuscule) importance in order to extract the most influential factor (see equation (2) in the paper by Ohnaka, 2004).

If all of the mechanisms suggested by Bizzarri and Cocco are considered and if the most fundamental and primary property is distinguished from the properties of secondary (or minuscule) importance, the constitutive law for the shear rupture has to be formulated in such a manner that the shear traction along the macroscopic rupturing surfaces degrades with ongoing slip in the breakdown zone. The reason for this is because the slip-dependency is a more fundamental and primary property of the shear rupture than any other property, including the rate-dependency (Ohnaka, 2004). In particular, I emphasize that it is common knowledge among experimentalists that the effect of slip rate is secondary to the primary effect of slip during the breakdown process; this has already been established in the laboratory. A recent laboratory experiment by Di Toro *et al.* (2004) indicated a remarkable reduction of steady-state frictional resistance of quartz rock at slip rates of the order of 1–10 cm/s. On the basis of this result, these researchers suggest that dynamic frictional resistance at seismic slip velocities of the order of 1 m/s may be extremely low (nearly zero). Note that this finding is compatible with the slip-dependent constitutive formulation.

It has often been argued that the rate- and state-dependent formulation is a more universal type of formulation than the slip-dependent formulation, because both the fault-healing (or re-strengthening) and–weakening processes can be treated by a single law within the framework of the rate- and state-dependent formulation. However, the rate- and state-dependent formulation has serious problems, as exemplified above (see also Ohnaka, 2004). Furthermore, one must recognize that the fault-healing process is a physical process that is completely different from the fault-weakening (or breakdown) process. Rigorously, there are no physical grounds that both processes must be governed by a single law (see Ohnaka, 2004). Mathematical convenience should not be a justification for simplifying the complex formulations of constitutive relations that govern different physical processes of fault-healing and fault-weakening. I emphasize that the underlying physics should be given priority over mathematical convenience in order for the physics of earthquakes to meet the criteria of a more complete, quantitative science.

The constitutive law that governs the behavior of earthquake ruptures provides the basis of earthquake physics, and the governing law plays a fundamental role in accounting for the entire process of an earthquake rupture, from its nucleation to its dynamic propagation to its arrest, quantitatively, in a unified and consistent manner. Therefore, in the absence of a rational law that governs real earthquake rupture processes, the physics of earthquakes cannot be a quantitative science (hopefully, an exact science) in the true sense of the definition. It is therefore of great urgency that such a rational law be established. Regrettably, however, it is still controversial what the governing law for earthquake ruptures ought to be, and how it should be formulated. Resolution of this controversy is a necessary step towards a more complete, unified theory of earthquake physics. With this aim, we need to discuss thoroughly in quantitative terms just what the governing law ought to be, on the basis of solid facts obtained from comprehensive viewpoints. In this sense, I appreciate this opportunity provided to reply the comment by A. Bizzarri and M. Cocco.

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